

Holographic Butterfly Effect at Quantum Critical Points

Yi Ling^{1,3,4,*}, Peng Liu^{1,†} and Jian-Pin Wu^{2,3‡}

¹ *Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*

² *Institute of Gravitation and Cosmology, Department of Physics,*

School of Mathematics and Physics, Bohai University, Jinzhou 121013, China

³ *Shanghai Key Laboratory of High Temperature Superconductors, Shanghai, 200444, China*

⁴ *School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China*

When the Lyapunov exponent λ_L in a quantum chaotic system saturates the bound $\lambda_L \leq 2\pi k_B T$, it is proposed that this system has a holographic dual described by a gravity theory. In particular, the butterfly effect as a prominent phenomenon of chaos can ubiquitously exist in a black hole system characterized by a shockwave solution near the horizon. In this letter we propose that the butterfly velocity v_B can be used to diagnose quantum phase transition (QPT) in holographic theories. We provide evidences for this proposal with two holographic models exhibiting metal-insulator transitions (MIT), in which the second derivative of v_B with respect to system parameters characterizes quantum critical points (QCP) with local extremes. We also point out that this proposal can be tested by experiments in the light of recent progress on the measurement of out-of-time-order correlation function (OTOC).

Introduction: The butterfly effect as chaotic behavior states that an initially small perturbation becomes non-negligible at later time. It can be diagnosed by the out-of-time-order correlation function (OTOC) in quantum systems. An OTOC of two generic Hermitian operators W, V is defined as

$$F(t, \vec{x}) = \frac{\langle W^\dagger(t, \vec{x}) V^\dagger(0, 0) W(t, \vec{x}) V(0, 0) \rangle_\beta}{\langle W(t, \vec{x}) W(t, \vec{x}) \rangle_\beta \langle V(0, 0) V(0, 0) \rangle_\beta}, \quad (1)$$

where $W(t, \vec{x}) \equiv e^{iHt} W(0, \vec{x}) e^{-iHt}$, and $\langle \cdots \rangle_\beta$ represents the ensemble average at temperature $T = 1/(k_B \beta)$. The OTOC diagnoses the butterfly effect by a sudden decay after the scrambling time t_* , which generically takes the following form,

$$F(t, \vec{x}) = 1 - \alpha e^{\lambda_L \left(t - t_* - \frac{|\vec{x}|}{v_B} \right)} + \cdots, \quad (2)$$

where v_B is the butterfly velocity and λ_L is the Lyapunov exponent. While the scrambling time t_* is the timescale when the commutator $[W(t, \vec{x}), V(0, 0)]$ grows to $\mathcal{O}(1)$. Physically, $F(t)$ describes the spread, or the scrambling of quantum information over the degrees of freedom across the system. Actually, the real part of $F(t)$ satisfies $\text{Re}[F(t, \vec{x})] = 1 - \langle ||[W(t, \vec{x}), V(0, 0)]||^2 \rangle_\beta / 2$, and the growth of the commutator $[W(t, \vec{x}), V(0, 0)]$ can diagnose the quantum information propagation [1]. Very importantly, as a characteristic velocity of a chaotic quantum system, v_B sets a bound on the speed of the information propagation [1]. Besides diagnosing chaotic behavior, the OTOC can be used to identify the many-body localization (MBL) phase [2].

Black holes have been providing a fascinating arena for the interplay of general relativity, condensed matter physics and quantum information. The development

in this direction becomes more exciting in recent years when the gauge/gravity duality, as a powerful tool for understanding the strongly correlated system, has rendered new insights into quantum critical phenomena and chaotic behaviors in many-body system [1, 3–5]. In particular, the butterfly effect has extensively been studied in this context [6–18]. In the study of high energy scattering near horizon and information scrambling of black holes it is found that the butterfly effect ubiquitously exists and is signaled by a shockwave solution near the horizon [1, 6, 7, 10, 16] (see also Appendix B). Especially, a bound on chaos has been proposed as

$$\lambda_L \leq \frac{2\pi}{\beta}, \quad (3)$$

and the saturation of this bound has been suggested as the criterion on whether a many-body system has a holographic dual with a bulk theory [11]. One remarkable example that saturates this bound is the Sachdev-Ye-Kitaev (SYK) model [11, 19]. Recently, the butterfly velocity v_B has also been conjectured as the characteristic velocity that universally bounds the diffusion constants in incoherent metal [16–18, 20].

On the other hand, metal-insulator transition (MIT) as a prominent style of quantum phase transition (QPT) has been implemented in holographic approach as well [5, 21–23]. In this scenario QPT occurs when the horizon is deformed to different IR fixed points which are separately dual to the ground states in metallic phase or insulating phase.

In this letter our main purpose is to connect the butterfly effect and QPT in holographic approach. Since in holographic theories the bound in (3) is always saturated, we will focus on the behavior of the butterfly velocity v_B close to quantum critical points (QCP)¹.

*Electronic address: lingy@ihep.ac.cn

†Electronic address: liup51@ihep.ac.cn

‡Electronic address: jianpinwu@mail.bnu.edu.cn

¹ Previously, it was demonstrated in [24] that the Lyapunov ex-

The first signal to connect v_B and QPT comes from the fact that both the butterfly velocity v_B and the phase transition are controlled by IR degrees of freedom in chaotic quantum system [1, 5]. This picture becomes more vivid in holographic scenario since IR degrees of freedom of the dual field theory is reflected by the near horizon data, and both v_B and QPT depend solely on the near horizon data. In addition, the OTOC exhibits butterfly effect for any operator that affects the energy of the bulk theory [1, 9, 19], while QPT is characterized by the degeneracy of ground states, implying that the butterfly effect should be sensitive to QPT since they involve energy fluctuations. Therefore, it is highly possible that the OTOC or the butterfly effect can capture the QPT in holographic theories.

A heuristic argument about the relation between v_B and QPT comes from the different behavior of the information propagation during the transition from a MBL phase to a thermalized phase. A quantum system in MBL phase does not satisfy the Eigenstate Thermalization Hypothesis (ETH), and the quantum information propagates very slowly [2, 25]. In thermalized phase, however, the information propagates much faster. In other words, the speed of information propagation probably works as an indicator of a MBL phase transition. Remind that the butterfly velocity bounds the speed of the quantum information propagation across the chaotic system, it is reasonable to expect that v_B may exhibit different behavior in distinct phases.

Inspired by above considerations, we propose that the butterfly velocity v_B can characterize the QPT in generic holographic theories. We will present evidences for this proposal with two holographic models involving MIT as an example of QPT, and demonstrate that the second derivatives of v_B with respect to system parameters do capture the QPT by showing local extremes near QCPs. Also, we point out possible experiments to test our proposal in laboratory.

The butterfly effect and the quantum phase transition:

Holographic Scheme - In this section, we will provide evidences to support our proposal by explicitly constructing holographic models. The basic scheme can be summarized as follows. In the context of gauge/gravity duality, we establish a holographic description for the quantity in condensed matter physics such that it can be computed in terms of the metric and other matter fields in the bulk. On one hand, the butterfly effect in black holes has been investigated in [1, 16, 26, 27], and the butterfly velocity can be extracted from shockwave solutions to the perturbation equations of gravity. We present the detailed derivation for v_B over an anisotropic bulk geometry in Appendix B. On the other hand, the holographic description of MIT has been studied in [5, 21–23], where the

lattice structure plays a key role in generating relevant deformations to near horizon geometry. In this letter we consider two holographic models exhibiting MIT, which have been presented in A 1 and A 2 (for more details of both holographic models, we refer to [21–23]). Moreover, one essential ingredient for distinguishing the metallic phase and insulating phase is the temperature behavior of DC conductivity σ_{DC} , which can be completely determined by the horizon data as well. We present the expression of σ_{DC} in Appendix A 3. Specifically, when a holographic model is specified, both σ_{DC} and v_B can be computed via Eqs.(A5), (A6) and (B9), respectively, where the data on horizon are obtained by numerically solving the background equations.

Numerical Results and Evidences - The system parameters in both holographic models are (λ, k) , corresponding to the lattice strength and wave number, respectively. Now our main task is to compute the butterfly velocity v_B for different backgrounds parameterized by (λ, k) . Fig.1 illustrates the behavior of v_B in model A 1 over the phase diagram at an extremely low temperature $T = 10^{-3}$. As one can see from the first plot of Fig. 1, the butterfly velocity itself v_B is featureless in the vicinity of the critical line (red curve), which denotes the location of MIT and is plotted by virtue of $\partial_T \sigma_{DC}(T) = 0$. In the second plot the local maxima of $\partial_k v_B$ are close to the critical line, while with a mild discrepancy. In the third plot, the local extremes of $\partial_k^2 v_B$ matches well with the critical line. Very similar results have been obtained for $\partial_\lambda v_B$ and $\partial_\lambda^2 v_B$. As a consequence, we arrive at the main result of our work, stating that the second derivative of the butterfly velocity v_B with respect to system parameters displays local extremes at the critical points.

Next we demonstrate that the phenomena of v_B characterizing QPT is robust in low temperature region. For a fixed value of λ , we change the parameter k and locate the local extreme $\tilde{k}_c(T)$ of $\partial_k^2 v_B$ and then denote its discrepancy with the critical point $k_c(T)$ by $\Delta k_c(T) \equiv \tilde{k}_c(T) - k_c(T)$. We observe the temperature behavior of this quantity and find it vanishes in zero temperature limit. For simplicity, we show an example in Fig. 2, which clearly indicates that the local extremes of $\partial_k^2 v_B$ match the QCPs in zero temperature limit. For other choices of parameters, similar phenomena can be obtained.

The butterfly velocity v_B has a power law dependence on temperature $v_B \sim T^\alpha$ in low temperature region [1, 16]. We now discuss the scaling behavior of v_B in the first model. We plot $\frac{T}{v_B} \frac{\partial v_B}{\partial T}$ versus T in Fig. 2, which numerically extracts the exponent α in scaling region. In low temperature region α is positive for both phases, which means that v_B goes to zero in zero temperature limit. Especially, in insulating phases α depends on parameters, while in metallic phases α fixes at 0.5. This reflects the fact that our metallic phase corresponds to AdS_2 IR geometry solely, which is also in accordance with the results in [16].

ponent λ_L may exhibit a peak near QCP in the Bose-Hubbard model.

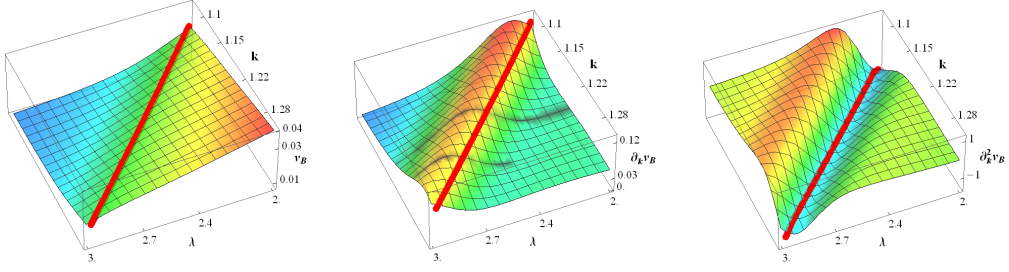


FIG. 1: Model A 1. 3D plots of v_B , $\partial_k v_B$, $\partial_k^2 v_B$ over the parameter space (λ, k) . The red curve in each plot is the critical line of MIT.

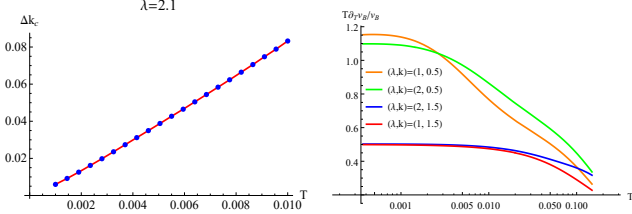


FIG. 2: Left plot: the temperature dependence of Δk_c . Right plot: $\frac{T}{v_B} \frac{\partial v_B}{\partial T}$ vs. T with different parameter values. The red and blue curves correspond to metallic phases while the orange and green curves correspond to insulating phases.

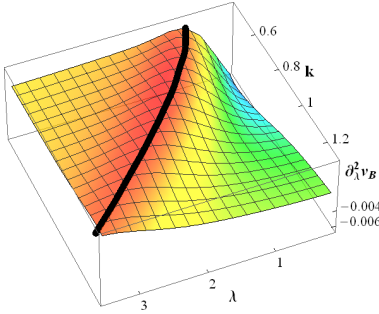


FIG. 3: Model A 2. 3D plot of $\partial_\lambda^2 v_B$ over parameter space (λ, k) . The black curve is the critical line of MIT.

Further evidence for our proposal can be found in the second model A 2, which does not contain AdS_2 geometry as IR fixed point. In this model, it is again the second derivative of v_B that characterizes the QCP. We demonstrate the behavior of $\partial_\lambda^2 v_B$ over the phase diagram at an extremely low temperature $T = 0.02$ in Fig. 3. In this figure a ridge forms in the vicinity of the critical line rather than a valley as shown in the previous model. Furthermore, we have also checked that v_B characterizing the QPT is robust in low temperature region.

Discussion and prospect for experiments - In above models we have demonstrated that the second derivative of v_B with respect to system parameters diagnoses the QCP with local extremes. As an extension we believe the scenario of v_B characterizing QPT is applicable to other holographic models with MIT (for instance the

isotropic lattice model, the lattice with helical symmetry or massive gravity), and also to other holographic models exhibiting other sorts of QPT. Of course in these circumstances the characterizing style of v_B can be more diverse.

Although our evidences comes from holographic theories that always saturate the chaos bound, our proposal may also apply for chaotic quantum system that does not saturate the bound. A direct argument is that IR dependence of v_B and QPT does not require a holographic theory.

More importantly, our proposal can be tested by experiments in light of recent progress on the measurement of OTOC. Experimentally, the butterfly velocity v_B , and its relation to QPT, can be studied by measuring the OTOC of a QPT system. Recently, new protocols and methods, that are versatile to simulate diverse many-body systems and achievable with state-of-the-art technology, have been proposed to measure the OTOC [15, 28]. Furthermore, experimental measurements of the OTOC have also been implemented [29, 30]. All these progresses provide test beds for our proposal.

Final Remarks: To sum up, we have proposed that the butterfly velocity v_B can characterize the QPT, and provided evidences with two holographic models exhibiting MIT. Furthermore, we have pointed out that our proposal may apply for quantum chaotic system that does not saturate the chaos bound.

One crucial issue arises in two models of this letter, namely, why the QPT is characterized by the *second* order of the derivative of v_B . We think the underlying reasons depend on how the IR degrees of freedom are encoded in the butterfly effect, or in holographic perspective, on how the quantity $\partial_{\lambda,k}^2 v_B$ is related to $\partial_T \sigma_{DC}$ exactly. A definite answer to this question need further investigation.

Our work has offered an information-theoretic diagnose of the QPT. The distinct behavior of information propagation in a quantum many-body system may signalize different phases. This phenomenon indicates that the information property of a chaotic many-body system can work as a novel tool to study QPT. It can be expected that more insights will be gained into QPT from the quantum information theory.

Acknowledgements - We are very grateful to Meng-

he Wu, Zhuo-yu Xian, Yi-kang Xiao and Xiang-rong Zheng for helpful discussion. This work is supported by the Natural Science Foundation of China under Grant Nos.11275208, 11305018 and 11575195, and by the grant (No. 14DZ2260700) from the Opening Project of Shanghai Key Laboratory of High Temperature Superconductors. Y.L. also acknowledges the support from Jiangxi young scientists (JingGang Star) program and 555 talent project of Jiangxi Province. J. P. Wu is also supported by the Program for Liaoning Excellent Talents in University (No. LJQ2014123).

Appendix A: Holographic setup

1. Holographic Q-lattice model

The Lagrangian of the holographic Q-lattice model reads as [21, 22, 31],

$$\mathcal{L} = R + 6 - \frac{1}{4}F^2 - |\nabla\Phi|^2 - m^2|\Phi|^2, \quad (\text{A1})$$

where $F = dA$ is the field strength of the Maxwell field and Φ is the complex scalar field simulating the Q-lattice structure. Note that we have set the AdS radius $L = 1$. The ansatz for a black brane solution with lattice structure only along x direction is presented as

$$ds^2 = \frac{1}{z^2} \left(-fSdt^2 + \frac{dz^2}{fS} + \hat{V}_x dx^2 + \hat{V}_y dy^2 \right),$$

$$A_t = \mu(1-z)a, \quad \Phi = e^{i\tilde{k}x} z^{3-\Delta} \phi, \quad (\text{A2})$$

where $f(z) \equiv (1-z)(1+z+z^2-\mu^2 z^3/4)$ and $\Delta = 3/2 \pm (9/4 + m^2)^{1/2}$. $S, \hat{V}_x, \hat{V}_y, a$ and ϕ are functions of the radial coordinate z only and μ is the chemical potential of the dual field theory. Black brane solutions are obtained by numerically solving the Einstein equations as well as other equations of motion for matter fields. Each solution is specified by three dimensionless parameters, namely the temperature \tilde{T}/μ with $\tilde{T} = (12 - \mu^2)S(1)/16\pi$, lattice amplitude $\tilde{\lambda}/\mu^{3-\Delta}$ with $\tilde{\lambda} \equiv \phi(0)$, and lattice wave number \tilde{k}/μ , which is abbreviated as $\{T, \lambda, k\}$ in this paper. The metric has an event horizon at $z = 1$ and the spacetime boundary locates at $z = 0$. Throughout this paper, we set $m^2 = -2$ such that the scaling dimension of Φ is $\Delta = 2$.

The occurrence of MIT in this model has been discussed in [21] and an explicit phase diagram over (λ, k) plane has been presented in [22], where the temperature is fixed at $T \sim 10^{-3}$, but further decreasing the temperature will not receive significant modifications. Note that in this model the near horizon geometry for a metallic phase is AdS_2 in zero temperature limit which contains non-zero entropy density.

2. Gubser-Rocha model with Q-lattice

The Lagrangian of the Gubser-Rocha model with Q-lattice is

$$\mathcal{L} = R - |\nabla\Phi|^2 - m^2|\Phi|^2 - \frac{1}{4}F^2 - \frac{e^{\Phi_1}}{4}G^2 - \frac{3}{2}|\nabla\Phi_1|^2 + 6\text{Cosh}(\Phi_1). \quad (\text{A3})$$

Compared with (A1), a real dilatonic field Φ_1 and an additional gauge field $G = dB$ are introduced. Note that here, $F = dA$ is the real Maxwell field being responsible for the electrical transport. The ansatz for metric and the complex scalar field takes the same form as (A2) but with $f(z) = (1-z)p(z)/g(z)$, $p(z) = 1 + (1+3Q)z + (1+3Q(1+Q))z^2$, and $g(z) = (1+Qz)^{3/2}$. While the two gauge fields and the dilatonic field take the following form

$$A_t(z) = \frac{(1-z)}{1+Qz}\mu a, \quad B_t(z) = \frac{(1-z)}{1+Qz}\mu b,$$

$$\Phi_1(z) = \frac{1}{2}\ln(1+Qz\phi_1), \quad (\text{A4})$$

where $\mu = \sqrt{3Q(1+Q)}$. Functions $(S, \hat{V}_x, \hat{V}_y, a, b, \phi, \phi_1)$ all depend on the radial direction z only. The Hawking temperature of the black brane is given by $\tilde{T} = 3\sqrt{1+Q}S(1)/4\pi$. The family of black brane solutions based on ansatz (A3) are now specified by four dimensionless quantities $\{T, \lambda, k, a_0\}$, where a_0 is from gauge field A and set as $a_0 = 0.1$ in our numerical calculation. This model is characterized by ground states with vanishing entropy density for both metallic and insulating phases [23].

3. DC conductivity and MIT

The expressions of DC conductivity σ_{DC} along x -direction for models A1 and A2 are given by

$$\sigma_{DC} = \left(\sqrt{\frac{\hat{V}_y}{\hat{V}_x}} + \frac{\mu^2 a^2 \sqrt{\hat{V}_x \hat{V}_y}}{k^2 \phi^2} \right) \Big|_{z=1}, \quad (\text{A5})$$

$$\sigma_{DC} = \left(\sqrt{\frac{\hat{V}_y}{\hat{V}_x}} + \frac{3Qa^2 \sqrt{\hat{V}_x \hat{V}_y}}{2k^2(1+Q)\phi} \right) \Big|_{z=1}. \quad (\text{A6})$$

They are determined by the horizon data [22, 23, 31].

At finite but extremely low temperature, we distinct the metallic phase and the insulating phase by the different temperature dependence of DC conductivity. Specifically, the metallic phase is defined by $\partial_T \sigma_{DC}(T) < 0$ while insulating phase $\partial_T \sigma_{DC}(T) > 0$, therefore the surface $\partial_T \sigma_{DC}(T) = 0$ separating the insulating phase and the metallic phase is the critical surface. This criterion has also been widely adopted in holographic literature [22, 23, 32].

Appendix B: The butterfly velocity

In this section we demonstrate the derivation of butterfly velocity v_B in anisotropic background, which can be extracted from the shockwave solution near the horizon [1, 16, 26, 27]. For this purpose, it is more convenient to work in r -coordinate with $r \equiv r_0/z$, where r_0 is the location of horizon. The generic spatially anisotropic metric of a 4-dimensional spacetime can be written as

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + V_x(r)dx^2 + V_y(r)dy^2. \quad (\text{B1})$$

In Kruskal coordinate (B1) is written as

$$ds^2 = \mathcal{U}(uv)dudv + V_x(uv)dx^2 + V_y(uv)dy^2, \quad (\text{B2})$$

where $uv = -e^{U'(r_0)r_*(r)}$, $u/v = -e^{-U'(r_0)t}$, with r_* being the tortoise coordinate defined by $dr_* = dr/U(r)$. In addition, $\mathcal{U}(uv) = \frac{4U(r)}{uvU'(r_0)^2}$, $V_{x,y}(uv) = V_{x,y}(r)$. Note that, in this coordinate the horizon is at $u = v = 0$.

The shockwave geometry is induced by a freely falling particle on the AdS boundary at t_i in the past and at $x = y = 0$. This particle is exponentially accelerated in Kruskal coordinate and generates the following energy distribution at $u = 0$,

$$\delta T_{uu} \sim E_0 e^{\frac{2\pi}{\beta} t_i} \delta(u) \delta(x, y), \quad (\text{B3})$$

where E_0 is the initial asymptotic energy of the particle. After the scrambling time $t_* \sim \beta \log N^2$ an initially small perturbation becomes significant and back-react to the geometry by a shockwave localized at the horizon [33],

$$ds^2 = V_x(uv)dx^2 + V_y(uv)dy^2 + \mathcal{U}(uv)dudv - \mathcal{U}(uv)\delta(u)h(x, y)du^2. \quad (\text{B4})$$

By a convenient redefinition $\tilde{y} \equiv y\sqrt{\frac{V_x(0)}{V_y(0)}}$ the resultant

Einstein equation can be written as a Poisson equation,

$$(\partial_x^2 + \partial_{\tilde{y}}^2 - m^2) h(x, \tilde{y}) \sim \frac{16\pi G_N V_x(0)}{\mathcal{U}(0)} E_0 e^{\frac{2\pi}{\beta} t_i} \delta(x, \tilde{y}), \quad (\text{B5})$$

with m^2 given by

$$m^2 = \frac{2}{\mathcal{U}(uv)} \left(V'_x(uv) + \frac{V_x(uv)V'_y(uv)}{V_y(uv)} \right) \Big|_{u=0}. \quad (\text{B6})$$

At long distance $|x| \geq m^{-1}$, the solution reads as

$$h(x, \tilde{y}) \sim \frac{E_0 e^{\frac{2\pi}{\beta}(t_i - t_*) - m|x|}}{|x|^{1/2}}. \quad (\text{B7})$$

From (B7) we read off the Lyapunov exponent λ_L and the butterfly velocity v_B ,

$$\lambda_L = \frac{2\pi}{\beta}, \quad v_B = \frac{2\pi}{\beta m} \quad (\text{B8})$$

The Lyapunov exponent saturates the chaos bound as expected. Rewriting m in coordinates (r, t, x, \tilde{y}) we find

$$v_B = \sqrt{\frac{2\pi T V_y(r_0)}{V'_x(r_0)V_y(r_0) + V_x(r_0)V'_y(r_0)}}. \quad (\text{B9})$$

When recovered to (x, y) coordinate system, the butterfly velocity \mathbf{v}_B is anisotropic. Specifically, in direction with polar angle θ ,

$$\mathbf{v}_B(\theta) = v_B \sqrt{\frac{\sec^2(\theta)V_y(r_0)}{V_y(r_0) + \tan^2(\theta)V_x(r_0)}}. \quad (\text{B10})$$

In this letter we focus on the butterfly velocity along x -direction, *i.e.*, $\mathbf{v}_B(0) = v_B$ in our work.

-
- [1] D. A. Roberts and B. Swingle, Phys. Rev. Lett. **117**, no. 9, 091602 (2016) [arXiv:1603.09298 [hep-th]].
 - [2] R. Fan, P. Zhang, H. Shen and H. Zhai, arXiv:1608.01914 [cond-mat.str-el].
 - [3] J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231, [arXiv:hep-th/9711200].
 - [4] E. Witten, Adv. Theor. Math. Phys. (1998) 253, [arXiv:hep-th/9802150].
 - [5] A. Donos and S. A. Hartnoll, Nature Phys. **9**, 649 (2013) [arXiv:1212.2998].
 - [6] S. H. Shenker and D. Stanford, JHEP **1403**, 067 (2014) [arXiv:1306.0622 [hep-th]].
 - [7] S. H. Shenker and D. Stanford, JHEP **1412**, 046 (2014) [arXiv:1312.3296 [hep-th]].
 - [8] D. A. Roberts, D. Stanford and L. Susskind, “Localized shocks,” JHEP **1503**, 051 (2015) [arXiv:1409.8180 [hep-th]].
 - [9] D. A. Roberts and D. Stanford, Phys. Rev. Lett. **115**, no. 13, 131603 (2015) [arXiv:1412.5123 [hep-th]].
 - [10] S. H. Shenker and D. Stanford, JHEP **1505**, 132 (2015) [arXiv:1412.6087 [hep-th]].
 - [11] J. Maldacena, S. H. Shenker and D. Stanford, JHEP **1608**, 106 (2016) [arXiv:1503.01409 [hep-th]].
 - [12] J. Polchinski, “Chaos in the black hole S-matrix,” arXiv:1505.08108 [hep-th].
 - [13] P. Hosur, X. L. Qi, D. A. Roberts and B. Yoshida, JHEP **1602** (2016) 004 [arXiv:1511.04021 [hep-th]].
 - [14] J. Polchinski and V. Rosenhaus, “The Spectrum in the Sachdev-Ye-Kitaev Model,” JHEP **1604**, 001 (2016) [arXiv:1601.06768 [hep-th]].
 - [15] B. Swingle, G. Bentsen, M. Schleier-Smith and P. Hayden, arXiv:1602.06271 [quant-ph].
 - [16] M. Blake, “Universal Charge Diffusion and the Butterfly Effect,” arXiv:1603.08510 [hep-th].

- [17] M. Blake, “Universal Diffusion in Incoherent Black Holes,” arXiv:1604.01754 [hep-th].
- [18] A. Lucas and J. Steinberg, “Charge diffusion and the butterfly effect in striped holographic matter,” arXiv:1608.03286 [hep-th].
- [19] A. Kitaev, “Hidden correlations in the hawking radiation and thermal noise,” (2014), talk given at the Fundamental Physics Prize Symposium, Nov. 10, 2014.
- [20] S. A. Hartnoll, “Theory of universal incoherent metallic transport,” Nature Phys. **11**, 54 (2015) [arXiv:1405.3651 [cond-mat.str-el]].
- [21] A. Donos and J. P. Gauntlett, JHEP **1404**, 040 (2014) [arXiv:1311.3292 [hep-th]].
- [22] Y. Ling, P. Liu, C. Niu, J. P. Wu and Z. Y. Xian, JHEP **1604**, 114 (2016) [arXiv:1502.03661 [hep-th]].
- [23] Y. Ling, P. Liu and J. P. Wu, Phys. Rev. D **93**, no. 12, 126004 (2016) [arXiv:1604.04857 [hep-th]].
- [24] H. Shen, P. Zhang, R. Fan and H. Zhai, arXiv:1608.02438 [cond-mat.str-el].
- [25] J. M. Deutsch, Phys. Rev. A **43**, 2146 (1991).
- [26] T. Dray and G. 't Hooft, Nucl. Phys. B **253**, 173 (1985).
- [27] Y. Kiem, H. L. Verlinde and E. P. Verlinde, Phys. Rev. D **52**, 7053 (1995) [hep-th/9502074].
- [28] N. Y. Yao, F. Grusdt, B. Swingle, M. D. Lukin, D. M. Stamper-Kurn, J. E. Moore and E. A. Demler, arXiv:1607.01801 [quant-ph].
- [29] M. Gärttner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger and A. M. Rey, arXiv:1608.08938 [quant-ph].
- [30] J. Li, R. H. Fan, H. Y. Wang, B. T. Ye, B. Zeng, H. Zhai, X. H. Peng and J. F. Du, arXiv:1609.01246 [cond-mat.str-el].
- [31] A. Donos and J. P. Gauntlett, JHEP **1406**, 007 (2014) [arXiv:1401.5077 [hep-th]].
- [32] M. Baggioli and O. Pujolas, Phys. Rev. Lett. **114**, no. 25, 251602 (2015) [arXiv:1411.1003 [hep-th]].
- [33] Y. Sekino and L. Susskind, JHEP **0810**, 065 (2008) [arXiv:0808.2096 [hep-th]].